Refined Beam and Plate Theories

by

R. P. Shimpi

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Bombay Powai, Mumbai – 400 076
India
Galileo Galilei
(1564 – 1642)
Galileo Galilei’s Attempts at Beam Theory
Jakob Bernoulli (1654 – 1705)
Leonhard Euler (1707-1783)
Daniel Bernoulli (1700 - 1782)
Stephen P. Timoshenko
(1878 - 1972)
Napoleon Bonaparte
(1769 - 1821)
Ernst Chladni
(1756 - 1827)
Chladni nodal patterns of guitar plates
One of the pioneers of elasticity theory, she won the grand prize from the Paris Academy of Sciences.

Marie-Sophie Germain
(1776 – 1831)
A. E. H. Love
(1863 - 1940)
G. R. Kirchhoff
(1824 – 1887)
Eric Reissner
(1913 - 1996)
R. D. Mindlin
(1906 – 1987)
In the context of thick plate:

• Classical plate theory is not accurate enough.

• Classical plate theory results serve as:
  - lower limiting ones in case of deflections, and
  - upper limiting ones in case of natural frequencies.
Important Contributors

Srinivas, S and Rao. A. K.

Pagano, N. J.

Krishna Murty, A. V.

Lo, Christensen and Wu

Levinson, M.

Reddy, J. N.

Kant, T.  (IIT-Bombay)
Fig. 1  Geometry of a plate
Refined Plate Theory and Its Variants

Rameshchandran P. Shimpil
Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India

The development of a new refined plate theory and its two simple variants is given. The theories have strong commonality with the equations of classical plate theory (CPT). However, unlike CPT, these theories assume that lateral and axial displacements have bending and shear components such that bending components do not contribute toward shear forces and, likewise, shearing components do not contribute toward bending moments. The theory and one of its variants are variationally consistent, whereas the second variant is variationally inconsistent and uses the relationships between moments, shear forces, and bending. It should be noted that, unlike any other refined plate theory, the governing equation as well as the expressions for moments and shear forces associated with this variant are identical to those associated with the CPT, save for the appearance of a subscript. The effectiveness of the theory and its variants is demonstrated through an example. Surprisingly, the answers obtained by both the variants of the theory, one of which is variationally consistent and the other one is inconsistent, are same. The numerical example studied, therefore, not only brings out the effectiveness of the theories presented, but also, albeit unintentionally, supports the doubts, first raised by Levinson, about the so-called superiority of variationally consistent methods.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>a</td>
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<td>b</td>
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<tr>
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<tr>
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<tr>
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<td>Cartesian coordinates</td>
</tr>
<tr>
<td>ξ, η, z</td>
<td>Cartesian coordinate system</td>
</tr>
<tr>
<td>σxy, σyz, σzx</td>
<td>shear strains</td>
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Introduction

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Refined Plate Theory and Its Variants

Rameshchandran P. Shimpil
Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India

The development of a new refined plate theory and its two simple variants is given. The theories have strong commonality with the equations of classical plate theory (CPT). However, unlike CPT, these theories assume that lateral and axial displacements have bending and shear components such that bending components do not contribute toward shear forces and, likewise, shearing components do not contribute toward bending moments. The theory and one of its variants are variationally consistent, whereas the second variant is variationally inconsistent and uses the relationships between moments, shear forces, and bending. It should be noted that, unlike any other refined plate theory, the governing equation as well as the expressions for moments and shear forces associated with this variant are identical to those associated with the CPT, save for the appearance of a subscript. The effectiveness of the theory and its variants is demonstrated through an example. Surprisingly, the answers obtained by both the variants of the theory, one of which is variationally consistent and the other one is inconsistent, are same. The numerical example studied, therefore, not only brings out the effectiveness of the theories presented, but also, albeit unintentionally, supports the doubts, first raised by Levinson, about the so-called superiority of variationally consistent methods.

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Introduction

P ATE analysis involving higher-order effects such as shear effects is an involved and tedious process. Even the considerably simple and well-known first-order shear deformation theories such as Reissner's theory1 and Mindlin's theory2 require solving two different equations involving two unknown functions and involve the use of shear coefficient to approximately satisfy the constitutive relationship between shear stress and shear strain. This coefficient itself is a matter of research even in case of beams.

However, it is possible to take into account the higher-order effects and yet keep the complexity at a considerably lower level. Noteworthy contributions in this respect are from Librescu3,4 Levinson5, and Donnel6. The present author's previous work7 may also be cited in this respect. A critical review of the classical plate theory (CPT) as well as well-known higher-order theories is given by Vasi7 of.

Librescu's approach8 makes the use of weighted lateral displacement. Constitutive relations between shear stresses and shear strains are satisfied. Reissner's formulation comes out as a special case of Librescu's approach.

Levinson's5 uses a strength of materials approach and his theory, unlike that of Mindlin2, does not require admissible considerations to achieve its results. The governing equations Levinson's for the motion of a plate are the same as those of Mindlin's, provided that the shear coefficient value associated with Mindlin's theory is taken as 2/3.

Donnel's8 approach is to make corrections to the classical plate deflections. He assumes that the shear forces are uniformly distributed across the thickness of the plate, and to rectify this assumption, introduces a numerical factor, which needs to be adjusted. Constitutive relations between transverse shear stresses and strains are not satisfied exactly.

The present author's previous work7 utilizes physically meaningful entities, for example, displacement and shear forces, for describing the displacement field. Gross equilibrium equations of the plate are utilized to get a fourth-order partial differential equation. The theory is variationally inconsistent but easy to use.

The purpose of this paper is to introduce a new variationally consistent refined plate theory and, more important, its two simple variants. One of the variants is variationally consistent, but the other one is inconsistent. Note that the theories have strong similarity with the CPT, with respect to appearances and forms of some equations and expressions. In fact, unlike any other refined plate theory, the governing equation as well as the expressions for moments and shear forces associated with the second variant of the theory are identical to those associated with the CPT, save for the appearance of a subscript.

For developing the theories, axial as well as lateral displacements are allowed to be also influenced by shear forces. A unique feature of the present work is that lateral and axial displacements have bending and shear components such that bending components do not contribute toward shear forces and, likewise, shearing components do not contribute toward bending moments. This results in simplification in formulation.

Note that Mindlin's formulation2 comes out as a special case of Levinson's formulation5 and Reissner's formulation1 comes out as a special case of Librescu's formulation8 whereas CPT comes out

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*Professor, Aerospace Engineering Department.
A Brief Look Into the Refined Plate Theory (RPT)
Assumptions In RPT

• Displacements have components: bending and shear
  - bending components do not contribute towards shear forces
  - shearing components do not contribute towards bending moments

• The displacements are small
Assumptions In RPT
continued ...

• Transverse normal stress is negligible compared to in-plane stresses

• The constitutive relations in respect of shear stresses are satisfied

• Shear stresses are zero at top and bottom and vary parabolically
Displacement Field of RPT

\[ u = -z \frac{\partial w_b}{\partial x} + h \left[ \frac{1}{4} \left( \frac{z}{h} \right) - \frac{5}{3} \left( \frac{z}{h} \right)^3 \right] \frac{\partial w_s}{\partial x} \]

\[ v = -z \frac{\partial w_b}{\partial y} + h \left[ \frac{1}{4} \left( \frac{z}{h} \right) - \frac{5}{3} \left( \frac{z}{h} \right)^3 \right] \frac{\partial w_s}{\partial y} \]

\[ w = w_b + w_s \]
Expressions for Moments in RPT

\[ M_x = -D \left( \frac{\partial^2 w_b}{\partial x^2} + \mu \frac{\partial^2 w_b}{\partial y^2} \right) \]

\[ M_y = -D \left( \frac{\partial^2 w_b}{\partial y^2} + \mu \frac{\partial^2 w_b}{\partial x^2} \right) \]

\[ M_{xy} = -D \left( 1 - \mu \right) \frac{\partial^2 w_b}{\partial x \partial y} \]

Where, Plate rigidity :

\[ D = \frac{E h^3}{12 \left( 1 - \mu^2 \right)} \]
Expressions for Shear Forces in RPT

\[\begin{align*}
Q_x &= \frac{5 E h}{12 (1 + \mu)} \frac{\partial w_s}{\partial x} \\
Q_y &= \frac{5 E h}{12 (1 + \mu)} \frac{\partial w_s}{\partial y}
\end{align*}\]
Total Potential Energy in RPT

\[ \pi = \frac{E \ h^3}{12 \ (1 - \mu^2)} \int_{y=0}^{y=b} \int_{x=0}^{x=a} \Phi_1 \ dx \ dy \]

\[ + \frac{5 E \ h}{12 \ (1 + \mu)} \int_{y=0}^{y=b} \int_{x=0}^{x=a} \Phi_2 \ dx \ dy \]

\[ + \frac{E \ h^3}{1008 \ (1 - \mu^2)} \int_{y=0}^{y=b} \int_{x=0}^{x=a} \Phi_3 \ dx \ dy \]

\[ - \int_{y=0}^{y=b} \int_{x=0}^{x=a} q \left[ w_b + w_s \right] \ dx \ dy \]
\[
\Phi_1 = \frac{1}{2} \left( \frac{\partial^2 w_b}{\partial x^2} \right)^2 + \frac{1}{2} \left( \frac{\partial^2 w_b}{\partial y^2} \right)^2 \\
+ \mu \frac{\partial^2 w_b}{\partial x^2} \frac{\partial^2 w_b}{\partial y^2} + (1 - \mu) \left( \frac{\partial^2 w_b}{\partial x \partial y} \right)^2
\]

\[
\Phi_2 = \frac{1}{2} \left( \frac{\partial w_s}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w_s}{\partial y} \right)^2
\]

\[
\Phi_3 = \frac{1}{2} \left( \frac{\partial^2 w_s}{\partial x^2} \right)^2 + \frac{1}{2} \left( \frac{\partial^2 w_s}{\partial y^2} \right)^2 \\
+ \mu \frac{\partial^2 w_s}{\partial x^2} \frac{\partial^2 w_s}{\partial y^2} + (1 - \mu) \left( \frac{\partial^2 w_s}{\partial x \partial y} \right)^2
\]
Governing Equations of RPT

\[ \nabla^2 \ \nabla^2 \ w_b = \frac{q}{D} \]

\[ \frac{1}{84} \left( \nabla^2 \ \nabla^2 \ w_s \right) - \frac{5}{h^2} \left( -\mu \right) \left( \nabla^2 w_s \right) = \frac{q}{D} \]

Where,

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]
Boundary Conditions in RPT

Corner conditions:

Boundary conditions involving $w_b$:

$$-D \left[ (1 - \mu) \frac{\partial^2 w_b}{\partial x \partial y} \right] = 0 \text{ or } w_b \text{ is prescribed}$$

Boundary conditions involving $w_s$:

$$-D \left[ (1 - \mu) \frac{\partial^2 w_s}{\partial x \partial y} \right] = 0 \text{ or } w_s \text{ is prescribed}$$
On edges $x = 0$ and $x = a$:

Boundary conditions involving $w_b$:

\[- D \left[ \frac{\partial^3 w_b}{\partial x^3} + (2 - \mu) \frac{\partial^3 w_b}{\partial x \partial y^2} \right] = 0 \quad \text{or} \quad w_b \text{ is prescribed}\]

\[- D \left[ \frac{\partial^2 w_b}{\partial x^2} + \mu \frac{\partial^2 w_b}{\partial y^2} \right] = 0 \quad \text{or} \quad \frac{\partial w_b}{\partial x} \text{ is prescribed}\]
Boundary Conditions in RPT

continued ...

On edges \( x = 0 \) and \( x = a \):

Boundary conditions involving \( w_s \) :

\[
- D \left[ \frac{\partial^2 w_s}{\partial x^2} + \mu \frac{\partial^2 w_s}{\partial y^2} \right] = 0 \quad \text{or} \quad \frac{\partial w_s}{\partial x} \text{ is prescribed}
\]

\[
\frac{420 (1 - \mu) D}{h^2} \frac{\partial w_s}{\partial x} - D \left[ \frac{\partial^3 w_s}{\partial x^3} + (2 - \mu) \frac{\partial^3 w_s}{\partial x \partial y^2} \right] = 0 \quad \text{or} \quad w_s \text{ is prescribed}
\]
Boundary Conditions in RPT

continued ...

On edges \( y = 0 \) and \( y = b \):

Boundary conditions involving \( \psi_b \):

\[
-D \left[ \frac{\partial^3 \psi_b}{\partial y^3} + \left( 2 - \mu \right) \frac{\partial^3 \psi_b}{\partial x^2 \partial y} \right] = 0 \quad \text{or} \quad \psi_b \text{ is prescribed}
\]

\[
-D \left[ \frac{\partial^2 \psi_b}{\partial y^2} + \mu \frac{\partial^2 \psi_b}{\partial x^2} \right] = 0 \quad \text{or} \quad \frac{\partial \psi_b}{\partial y} \text{ is prescribed}
\]
Boundary Conditions in RPT

continued ...

On edges \( y = 0 \) and \( y = b \):

Boundary conditions involving \( w_s \):

\[
- D \left[ \frac{\partial^3 w_s}{\partial y^3} + 2 - \mu \right] \frac{\partial^3 w_s}{\partial x^2 \partial y} = 0 \text{ or } w_s \text{ is prescribed}
\]

\[
- D \left[ \frac{\partial^2 w_s}{\partial y^2} + \mu \frac{\partial^2 w_s}{\partial x^2} \right] = 0 \text{ or } \frac{\partial w_s}{\partial y} \text{ is prescribed}
\]
RPT Variant-I is arrived at ignoring insignificant terms in total potential energy expression of RPT
Expressions for Moments in RPT-Variant I

\[ M_x = -D \left( \frac{\partial^2 w_b}{\partial x^2} + \mu \frac{\partial^2 w_b}{\partial y^2} \right) \]

\[ M_y = -D \left( \frac{\partial^2 w_b}{\partial y^2} + \mu \frac{\partial^2 w_b}{\partial x^2} \right) \]

\[ M_{xy} = -D \left( 1 - \mu \right) \frac{\partial^2 w_b}{\partial x \partial y} \]

Where, Plate rigidity :

\[ D = \frac{E h^3}{12 \left( 1 - \mu^2 \right)} \]
Expressions for Shear Forces in RPT-Variant I

\[ Q_x = \frac{5 \ E \ h}{12 \ (1 + \mu)} \ \frac{\partial w_s}{\partial x} \]

\[ Q_y = \frac{5 \ E \ h}{12 \ (1 + \mu)} \ \frac{\partial w_s}{\partial y} \]
Total Potential Energy in RPT-Variant I

\[ \pi \approx \frac{E \ h^3}{12 \ (1 - \mu^2)} \int \int_{y=0 \atop x=0} \chi_1 \ dx \ dy \]

\[ + \frac{5 \ E \ h}{12 \ (1 + \mu)} \int \int_{y=0 \atop x=0} \chi_2 \ dx \ dy \]

\[ - \int \int_{y=0 \atop x=0} q \ [ w_b + w_s ] \ dx \ dy \]
\[ \chi_1 = \frac{1}{2} \left( \frac{\partial^2 w_b}{\partial x^2} \right)^2 + \frac{1}{2} \left( \frac{\partial^2 w_b}{\partial y^2} \right)^2 \\
+ \mu \left( \frac{\partial^2 w_b}{\partial x^2} \right) \frac{\partial^2 w_b}{\partial y^2} + \left( 1 - \mu \right) \left( \frac{\partial^2 w_b}{\partial x \partial y} \right)^2 \]

\[ \chi_2 = \frac{1}{2} \left( \frac{\partial w_s}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w_s}{\partial y} \right)^2 \]
Governing Equations of RPT Variant-I

\[ \nabla^2 \nabla^2 w_b = \frac{q}{D} \]

\[ \nabla^2 w_s = - \left[ \frac{h^2}{5 (1 - \mu)} \right] \left( \frac{q}{D} \right) \]
Corner condition:

\[-D \left[ (1 - \mu) \frac{\partial^2 w_b}{\partial x \partial y} \right] = 0 \quad \text{or} \quad w_b \text{ is prescribed}\]
Boundary Conditions in RPT-Variant I
continued ...

On edge $x = 0$ and $x = a$

Boundary conditions involving $w_b$:

\[ -D \left[ \frac{\partial^3 w_b}{\partial x^3} + \left( 2 - \mu \right) \frac{\partial^3 w_b}{\partial x \frac{\partial}{\partial y^2} } \right] = 0 \quad \text{or} \quad w_b \text{ is prescribed} \]

\[ -D \left[ \frac{\partial^2 w_b}{\partial x^2} + \mu \frac{\partial^2 w_b}{\partial y^2} \right] = 0 \quad \text{or} \quad \frac{\partial w_b}{\partial x} \text{ is prescribed} \]
Boundary Conditions in RPT-Variant I continued ...

On edge \( x = 0 \) and \( x = a \)

Boundary conditions involving \( u_s \):

\[
\frac{\partial u_s}{\partial x} = 0 \quad \text{or} \quad u_s \text{ is prescribed}
\]
Boundary Conditions in RPT-Variant
continued...

On edges $y = 0$ and $y = b$:

Boundary conditions involving $w_b$:

\[-D \left[ \frac{\partial^3 w_s}{\partial y^3} + (2 - \mu) \frac{\partial^3 w_s}{\partial x^2 \partial y} \right] = 0 \quad \text{or} \quad w_b \text{ is prescribed}\]

\[-D \left[ \frac{\partial^2 w_s}{\partial y^2} + \mu \frac{\partial^2 w_s}{\partial x^2} \right] = 0 \quad \text{or} \quad \frac{\partial w_b}{\partial y} \text{ is prescribed}\]
Boundary Conditions in RPT-Variant continued...

On edge $y = 0$ and $y = b$

Boundary conditions involving $w_s$:

$$\frac{\partial w_s}{\partial y} = 0 \quad \text{or} \quad w_s \text{ is prescribed}$$
RPT Variant-II is obtained by using gross equilibrium equations, instead of using principle of virtual work
Expressions for Moments in
RPT-Variant II

\[
M_x = -D \left( \frac{\partial^2 w_b}{\partial x^2} + \mu \frac{\partial^2 w_b}{\partial y^2} \right)
\]

\[
M_y = -D \left( \frac{\partial^2 w_b}{\partial y^2} + \mu \frac{\partial^2 w_b}{\partial x^2} \right)
\]

\[
M_{xy} = -D \left( 1 - \mu \frac{\partial^2 w_b}{\partial x \partial y} \right)
\]
Expressions for Shear Forces in RPT-Variant II

\[ Q_x = -D \frac{\partial}{\partial x} \left( \frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2} \right) \]

\[ Q_y = -D \frac{\partial}{\partial y} \left( \frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2} \right) \]

Where, Plate rigidity:

\[ D = \frac{E h^3}{12 \left(1 - \mu^2 \right)} \]
Governing Equation of RPT - Variant-II

∇² ∇² \( \varpi_b \) = \( \frac{q}{D} \)
Boundary Conditions in RPT-Variant II

If edge $x=a$ is simply supported:

$$W_{x=a} = 0, \quad M_{x=x=a} = 0$$
Boundary Conditions in RPT-Variant II
continued ...

If edge \( x=a \) is free:

\[
\begin{align*}
[M_{x \ x=a}^- &= 0, \\
\left[ Q_x + \frac{\partial M_{xy}}{\partial y} \right]_{x=a} &= 0
\end{align*}
\]
Boundary Conditions in RPT-Variant II continued ...

If edge $x=a$ is clamped:

\[ w_{,ab} = 0, \quad \left[ \frac{\partial w}{\partial x} \right]_{x=a} = 0 \]

OR

\[ w_{,x=a} = 0, \]

\[ \left[ \frac{\partial w_b}{\partial x} \right]_{x=a} = -\frac{3(1+\mu)}{E h} \left[ \frac{\partial}{\partial x} \left( \frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial x^2} \right) \right]_{x=a} \]
## Comparison of results for central deflection (at \( x = 0.5 \) and \( y = 0.5 \))

<table>
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<tr>
<th>Theory</th>
<th>Central deflection ( w )</th>
<th>Percentage Error with respect to exact theory</th>
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<tbody>
<tr>
<td>RPT</td>
<td>296.0568 ( q_o h / E )</td>
<td>0.6183</td>
</tr>
<tr>
<td>RPT-Variant I</td>
<td>296.0568 ( q_o h / E )</td>
<td>0.6219</td>
</tr>
<tr>
<td>RPT-Variant II</td>
<td>296.0568 ( q_o h / E )</td>
<td>0.6219</td>
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<tr>
<td>ZSDT for plate</td>
<td>296.0568 ( q_o h / E )</td>
<td>0.6219</td>
</tr>
<tr>
<td>CPT</td>
<td>280.2613 ( q_o h / E )</td>
<td>-4.7500</td>
</tr>
<tr>
<td>Exact plate theory</td>
<td>294.2375 ( q_o h / E )</td>
<td>0.0000</td>
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**Comparison of results for maximum tensile flexural stress $\sigma_x$ (at $x = 0.5$, $y = 0.5$, $z = 0.05$)**

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<tr>
<th>Theory</th>
<th>Maximum tensile flexural stress $\sigma_x$</th>
<th>Percentage Error with respect to exact theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPT</td>
<td>19.94322 $q_o$</td>
<td>- 0.5041</td>
</tr>
<tr>
<td>RPT-Variant I</td>
<td>19.94334 $q_o$</td>
<td>- 0.5035</td>
</tr>
<tr>
<td>RPT-Variant II</td>
<td>19.94334 $q_o$</td>
<td>- 0.5035</td>
</tr>
<tr>
<td>ZSDT for plate</td>
<td>19.94334 $q_o$</td>
<td>- 0.5035</td>
</tr>
<tr>
<td>CPT</td>
<td>19.75763 $q_o$</td>
<td>- 1.4300</td>
</tr>
<tr>
<td>Exact plate theory</td>
<td>20.04426 $q_o$</td>
<td>0.000</td>
</tr>
</tbody>
</table>
# Comparison of results for maximum shear stress $\tau_{zx}$ (at $x = 0$, $y = 0.5$, $z = 0$)

<table>
<thead>
<tr>
<th>Theory</th>
<th>Maximum shear stress $\tau_{zx}$</th>
<th>Percentage Error with respect to exact theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPT</td>
<td>$2.385722 \ q_o$</td>
<td>0.0000</td>
</tr>
<tr>
<td>RPT-Variant I</td>
<td>$2.387324 \ q_o$</td>
<td>0.0671</td>
</tr>
<tr>
<td>RPT-Variant II</td>
<td>$2.387324 \ q_o$</td>
<td>0.0671</td>
</tr>
<tr>
<td>ZSDT for plate</td>
<td>$2.387324 \ q_o$</td>
<td>0.0671</td>
</tr>
<tr>
<td>CPT</td>
<td>$2.387324 \ q_o$</td>
<td>0.0671</td>
</tr>
<tr>
<td>Exact plate theory</td>
<td>Not available</td>
<td>-----</td>
</tr>
</tbody>
</table>
Concluding Remarks

• Present theories are free from correction factor

• Transverse shear stresses and shear strains satisfy constitutive relations

• Results obtained by both the variants are same

• Finite element based on these theories will be free from shear locking

• The theories are easy to use
New First-Order Shear Deformation Plate Theories

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First-order shear deformation theories, one proposed by Reissner and another one by Mindlin, are widely in use, even today, because of their simplicity. In this paper, two new displacement based first-order shear deformation theories involving only two unknown functions, as against three functions in case of Reissner’s and Mindlin’s theories, are introduced. For static problems, governing equations of one of the proposed theories are uncoupled. And for dynamic problems, governing equations of one of the theories are only inertially coupled, whereas those of the other theory are only elastically coupled. Both the theories are variationally consistent. The effectiveness of the theories is brought out through illustrative examples. One of the theories has striking similarity with classical plate theory. [DOI: 10.1115/1.2423036]
Thank You