Improved Bounds for Policy Iteration in Markov Decision Problems

Shivaram Kalyanakrishnan

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Indian Institute of Technology Bombay
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November 2017

Collaborators: Neeldhara Misra, Aditya Gopalan, Utkarsh Mall, Ritish Goyal, Anchit Gupta
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Sequential Decision Making

AGENT

Think

Sense → Act

ENVIRONMENT

state reward action

Shivaram Kalyanakrishnan (2017) Analysis of Policy Iteration in MDPs
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https://img.tradeindia.com/fp/1/524/panoramic-elevators-564.jpg
Sequential Decision Making

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Shivaram Kalyanakrishnan (2017) Analysis of Policy Iteration in MDPs
Markov Decision Problems (MDPs)

\begin{align*}
&\text{s}_1 \quad 0.5, 0 \quad 0.25, -1 \\
&\quad \text{s}_2 \\
&\quad \quad 0.5, -1 \\
&\quad 1, 2 \\
&\quad 0.75, -2 \\
&\quad \text{s}_3 \\
&\quad 0.5, 3 \\
&\quad 0.5, 3 \\
&1, 1 \\
&1, 1 \\
&1, 1
\end{align*}
Markov Decision Problems (MDPs)

Elements of an MDP
- States ($S$)
- Actions ($A$)
- Transition probabilities ($T$)
- Rewards ($R$)

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$V^\pi$ is the **Value Function** of $\pi$. For $s \in S$,

$$V^\pi(s) = \mathbb{E}_\pi \left[ r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots | \text{start state} = s \right].$$
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Optimal Policies

$V^\pi$ satisfies a recursive equation: $V^\pi = R^\pi + \gamma T^\pi V^\pi$, which gives $V^\pi = (I - \gamma T^\pi)^{-1} R^\pi$. 
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What is the complexity of computing an optimal policy?
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One extra definition needed: **Action Value Function** \( Q^\pi_a \) for \( a \in A \).
\[
Q^\pi_a = R_a + \gamma T_a V^\pi.
\]
Given \( \pi \), a polynomial computation yields \( V^\pi \) and \( Q^\pi_a \) for \( a \in A \).
Policy Improvement
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Policy Improvement

\[ Q^\pi(s_7) > Q^\pi(s_7) \]

\[ Q^\pi(s_3) \leq Q^\pi(s_3) \]
Policy Improvement

Improvable states
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Improvable states

Improving actions
Given $\pi$, pick one or more improvable states, and in them, switch to an arbitrary improving action. Let the resulting policy be $\pi'$. 

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**Policy Improvement Theorem (H60, B12):**

1. If $\pi$ has no improvable states, then it is optimal, else
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\textbf{While} \( \pi \) has improvable states:

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Different **switching strategies** lead to different routes to the top.

**How long are the routes?!**
Switching Strategies and Bounds

### Upper bounds on number of iterations

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$\Omega(n)$ Howard’s PI on $n$-state, 2-action MDPs \[HZ10\].
Switching Strategies and Bounds

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<td>$1.6479^n$</td>
<td>$k^{0.7207n}$</td>
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Recursive Simple Policy Iteration

π
S1  S2  S3  S4  S5  S6  S7  S8
Recursive Simple Policy Iteration

Given $\pi$, pick the improvable state with the highest index, and, switch to an improving action picked uniformly at random. Let the resulting policy be $\pi'$. 
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Diagram: 
- States labeled $s_1$ to $s_8$. 
- States $s_1$, $s_2$, $s_3$, $s_4$, $s_5$, $s_6$, $s_7$, $s_8$. 
- States $s_7$, $s_8$ are highlighted as improvable states.

Shivaram Kalyanakrishnan (2017) 
Analysis of Policy Iteration in MDPs
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Policy Iteration: *widely used* algorithm, more than half a century old. Substantial *gap* exists between upper and lower bounds. We furnish several *exponential improvements* to upper bounds.
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Bears similarity to Simplex algorithm for Linear Programming. Howard’s PI works much better in practice than the variants for which we have shown improved upper bounds!

Open problem: Is the number of iterations taken by Howard’s PI on $n$-state, 2-action MDPs upper-bounded by the $(n + 2)$-nd Fibonacci number?
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For references see **tutorial**.

Theoretical Analysis of Policy Iteration Tutorial at IJCAI 2017
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