Algebra and Number Theory

Distinguished representations

A natural and important question in representation theory is to understand how a representation of a group restricts to a given subgroup. It was realized, almost three decades ago, that closely related questions in the context of p-adic and adelic groups have significant number theoretic aspects. In the case of p-adic groups, one is interested in finding out which representations of a group admit non-trivial linear forms that are invariant under the fixed group of an involution. Of much interest is also the local multiplicity question: what is the dimension of the space of invariant linear forms? In the adelic situation, the interest is in a specicific invariant form, called the period integral. And the right question is to ask when the period integral is non-vanishing. The work of Harder, Langlands, and Rapoport, in the mid eighties, on the Tate conjecture, as well as the philosophy, due to Jacquet, that the representations admitting non-trivial invariant forms (or non-vanishing period integrals in the global situation) often characterize the image of Langlands functorial lifts gave an impetus to the study of these representations, called distinguished representations. Distinguished representations and the related multiplicity questions are the central objects of study in the Relative Langlands Program promulgated by the recent works of Gan-Gross-Prasad-Waldspurger and Sakellaridis-Venkatesh. I am interested in certain aspects of the Relative Langlands Program.

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Schur-Weyl duality for algebraic group

I am interested in Schur-Weyl duality for algebraic groups over fields of positive characteristic. This involves computing the images of group algebras under a representation and checking if these images satisfy the double commutant properties. One then hopes to use this information in decomposing into irreducibles (or in computing the Jordan-Holder series of) the given representation and its tensor powers.

Prof. Shripad M. Garge, shripad@math.iitb.ac.in, smgarge@gmail.com

Quotients of smooth Z-homology 3-folds modulo (C,+) action are smooth

Following results were proved:

(1) For any non-trivial action of $(\mathbb{C},+)$ on the polynomial ring $\mathbb{C}[X,Y,Z]$ the ring of invariants $\mathbb{C}[X,Y,Z]^{G_a}$ is isomorphic to a polynomial ring in two variables.

(2) (Jointly with Avinash Sathaye)
Let $(\mathbb{C},+)$ act regularly and non-trivially on a smooth affine 3-fold X with trivial integral homology. Then the quotient $X/(\mathbb{C},+)$ is smooth.
The proof of (1) is much more accessible than the original proof of M. Miyanishi.

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The numbers game in commutative algebra

My research interests lie in Commutative and Homological Algebra, the study of commutative rings and modules over them. The problems I have worked on include understanding the rings and their modules using various numerics. In the past, I have studied approximating Artinian rings by Gorenstein rings via the notion of Gorenstein colength and studying new constructions of Gorenstein rings like connected sums.

I am currently involved in investigating various properties of fibre products and connected sums of rings, giving applications to rationality of Poincare Series. The study of Betti numbers and Poincare series has led me to an important area of current research, namely Boij-Soderberg Theory. Other ongoing projects include the study of Hilbert Series and multiplicities, including a generalisation of Lech's inequality.

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Existence of unimodular elements in a projective module

Let A be a commutative Noetherian ring and P be a finitely generated projective A-module. We say that P has a unimodular element or P splits off a free summand of rank 1 if P is isomorphic to $A \oplus Q$. If $A = R[\underline{X}, \underline{Y}^{\pm 1}]$ is Laurent polynomial ring and rank of P is $> d =$ dimension of R, then it is well known that P has a unimodular element. In each case below P has a unimodular element.

- If A is Laurent polynomial ring over $R[T, f^{-1}]$ with f a monic polynomial and rank $(P) > d$ (with Husney Parvez Sarwar, Journal of commutative Algebra).
- If $A = R[M]$ with $M \subset \mathbb{Z}_+^2$ a normal monoid of rank 2 and rank $(P) > d$ (with Husney Parvez Sarwar, Proc. Indian Acad. Sci. (Math. Sci.).
- If $A = R[X_1, \ldots, X_n]$, rank $(P) = d$ and P_f has unimodular element with f a monic polynomial in X_n (with Md. Ali Zinna, Preprint).
- If $A = R[X_1, \ldots, X_n]$, rank $(P) = d$ is even, $P/(X_1, \ldots, X_n)P$ has a unimodular element and $\exists$ a surjection $P \to I$ where I is an height n ideal generated by n elements (with Md. Ali Zinna, Preprint).

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Blow-up algebra's, asymptotic primes, local cohomology and AR-sequences

The broad area of my research is commutative algebra. I also use some non-commutative algebra which is useful in my work. I work in the following four areas:

- *Blow-up algebra's :* I have discovered an intruiging method to study associated graded rings. This involves the study of an infinitely generated module. I have discovered several applications of this method. I have already published four papers using this method. Two more papers are in the pipeline. *Asymptotic associate primes :* This is a bit technical subject. However this topic has lot of applications in other parts of commutative algebra (for instance in the study of blow-up algebras). Recently I have used techniques in the study of asymptotic primes. It leads us to an interesting conjecture, in the sense that if the conjecture is true then it is a good result, However if our conjecture is false then it is even more interesting.

- *Local cohomology over regular rings :* Local cohomolgy modules are usually intractable. However in the mid 1990's it was observed that local cohomology modules over regular rings behave much better. When we consider polynomial rings over the complex numbees then we can use theory of *D*-modules which shows that local cohomology modules are holonomic, In a recent work I have shown that natural constructions like Tor, Ext of graded local cohomolgy modules are special *D*-modules (they are generalized Eulerian).

- *AR-sequences :* The study of AR-sequences was first used in theory of (non-commutative) Artin algebra's. It has now used in the representation theory of Cohen-Macaulay isolated singularities. Using this technique recently I have shown that practically most two dimaensional isolated singularities have an infinite number of indecomposable self-dual representations of arbitrary rank.
- *Asymptotic associate primes :* This is a bit technical subject. However this topic has lot of applications in other parts of commutative algebra (for instance in the study of blow-up algebras). Recently I have used techniques used in the study of blow-up algebra's to investigate asymptotic primes. It leads us to an interesting conjecture, in the sense that if the conjecture is true then it is a good result, However if our conjecture is false then it is even more interesting.

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The analytic properties of L-functions associated to automorphic forms

My primary interests centre around characterising L-functions by means of their analytic properties and in establishing the desired analytic properties for L-functions associated to automorphic forms. Recent work has involved the exterior square L-functions associated to GL_n (over local and global fields) and primitivity results for the L-functions of cusp forms associated to GL_3 in the extended Selberg class.

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Blow-up algebras, Hilbert functions and local cohomology

My current research focuses on Hilbert functions of admissible filtrations of ideals in Cohen-Macaulay rings. These rings are central objects of study in commutative algebra. Through the knowledge of their Hilbert polynomials we can obtain information about various blow-up algebras such as Rees algebras, associated graded rings and the ring itself. Information about these algebras is often useful in resolution of singularities as demonstrated in the works of Abhyankar, Hironaka, Sally, Zariski and others.

We focus on their Cohen-Macaulay property. If a blow-up algebra is Cohen-Macaulay then computation of their algebraic and geometric invariants becomes easy since several homology and cohomology modules associated with them vanish. One of the techniques we use is the analysis of local cohomology modules of blow-up algebras.

Local cohomology modules were introduced by A. Grothendieck in the fifties. These modules have useful information which helps us in understating Hilbert polynomials as well as the Blow-up algebras. Specifically we use an avatar of the famous formula of Serre which gives difference of Hilbert function and the Hilbert polynomial in terms of the Euler Characteristic of all the local cohomology modules.

Our recent research using this line of attack has yielded unified proofs of several classical and modern results about completeness of products of complete ideals in regular local rings proved by Zariski, Lipman-Teissier, Rees and Reid-Roberts-Vitulli, The work of Reid-Roberts-Vitulli used convex geometry and hence is applicable to only polynomial rings. We have been able to isolate a homological obstruction in terms of Castelnuovo-Mumford regularity which enables us to find a general result in all dimensions in analytically unratified local rings which yields a unified proof of several earlier theorems.

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Subnormal operators

Study of special classes of operators on Hilbert spaces such as subnormal operators using Complex Analysis and Homological Algebra.

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Dilation of pair of commuting contractions

My current research area of interest is operator theory. To be more precise, my most recent works are based on dilation of operators. Dilation is a mathematical tool which largely used in different branches of Mathematics to understand the underlying object better. With no exception, dilation of operators is used to understand operators which are not normal. A brief summary of my recent works in this area are given below.

- Unitary dilation of a single contraction is the celebrated result due to Sz.-Nagy and Foias. In the case of pair of commuting contractions, T. Ando showed that they can be dilated to a pair of commuting unitaries. In my recent work with one of my collaborators, we obtain an another explicit method for finding unitary dilation of pair of commuting contractions such that at least one of the contractions is pure. Using this, we establish that the von-Neumann inequality for a pair of commuting pure contractions with finite defect spaces holds over distinguished varieties of the bidisc. This generalizes a result of Agler and McCarthy that for a pair of strict commuting matrices.
- Using the dilation method mentioned above, we also find a way to dilate pure pair of commuting operators to pure pair of isometries. In other words, a model for pure pair of commuting contractions is also obtained. This in turn provides a necessary and sufficient condition for a pure contraction to be product of two contractions.

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Invariants for liftings of operator tuples and its applications

Based on a careful analysis of functional models for contractive multi-analytic operators we have established a one-to-one correspondence between unitary equivalence classes of minimal contractive liftings of a row contraction and injective symbols of contractive multi-analytic operators. This allows an effective construction and classification of all such liftings with given defects. G. Popescu's theory of characteristic functions of completely non-coisometric row contractions is obtained as a special case satisfying a Szegö condition. In another special case of single contractions and defects equal to 1 all non-zero Schur functions on the unit disk appear in the classification. It is also shown that the process of constructing liftings iteratively reflects itself in a factorization of the corresponding symbols.

An application of these multi-analytic function was developed for quantum systems which we illustrate now: Using a scheme involving a lifting of a row contraction, we introduced a toy model of repeated interactions between quantum systems. In this model, there is an outgoing Cuntz scattering system involving two wandering subspaces. We associate to this model, an input/output linear system which leads to a transfer function. This transfer function is a multi-analytic operator, and we showed that it is inner if we assume that the system is observable. Finally it is established that transfer functions coincide with characteristic functions of associated liftings.

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CPD-kernels, k-families and Bures distance

We introduced, for any set S, the concept of $\mathfrak{K}$-family between two Hilbert C^*-modules over two C^*-algebras, for a given completely positive definite (CPD-) kernel $\mathfrak{K}$ over S between those C^*-algebras and obtained a factorization theorem for such $\mathfrak{K}$-families. If $\mathfrak{K}$ is a CPD-kernel and E is a full Hilbert C^*-module, then any $\mathfrak{K}$-family which is covariant with respect to a dynamical system (G, η, E) on E, extends to a $\tilde{\mathfrak{K}}$-family on the crossed product $E \times_\eta G$, where $\tilde{\mathfrak{K}}$ is a CPD-kernel. Several characterizations of $\mathfrak{K}$-families, under the assumption that E is full, were obtained and covariant versions of these results were also given. One of these characterizations says that such $\mathfrak{K}$-families extend as CPD-kernels, between associated (extended) linking algebras, whose $(2, 2)$-corner is a homomorphism and vice versa. This leads to new insights in the dilation theory of CPD-kernels in relation to $\mathfrak{K}$-families. Given an automorphism α on a C^*-algebra, we obtained a Kolmogorov decomposition of α-completely positive definite kernels (or α-CPD-kernels for short) and investigated a notion of distance between α-CPD-kernels called the Bures distance between α-CPD-kernels. We also defined transition probability for these kernels and found a characterization of the transition probability.

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Approximate solutions of integral equations and of associated Eigen value problems

The theory and application of integral equations is an important subject within applied mathematics. Integral equations are used as mathematical models for many physical situations.

I am interested in a numerical solution of the Fredholm integral equation of the second kind. The integral operator T is either a linear integral operator or a nonlinear Urysohn integral operator. The eigenvalue problem associated with a linear integral operator T is of equal interest.

As the exact solution can be obtained rarely, in order to find an approximate solution, the integral operator T is replaced by a finite-rank operator T_n. The integral equation associated with T_n can then be reduced to a finite system of linear/non-linear equations and the associated eigenvalue problem is reduced to a matrix eigenvalue problem, which can be solved using a computer. There are two principal approaches to construct a sequence of finite rank operators T_n approximating the integral operator. The first approach is to replace the integral by a convergent quadrature formula. This gives rise to the Nystrom operator. The other approach is based on a sequence of projection operators converging to the Identity operator pointwise. The classical Galerkin method and its variants fall under this category. In order to prove the convergence and the rate of convergence of the approximate solution to the exact solution, results from Functional Analysis are used.

My research work involves the theoretical work such as obtaining the orders of convergence for various methods and validating the theoretical results by implementation in specific cases using a computer. I have proposed a method called Modified Projection Method which improves the order of convergence significantly as compared to the Galerkin method. This improvement is achieved while retaining the computational complexity essentially the same as in the case of Galerkin method.
Currently I am working on integral equations with non-smooth kernels and on singular integral equations.

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Analysis of the Wu metric on Thullen domains in C^n

H. Wu constructed a new Hermitian metric that provides an interesting interpolant between the Finsler (like the Carathéodory and Kobayashi metrics) and the Kähler invariant metrics. The Wu metric raises several intriguing questions some of which I investigated. One of them is closely related to a problem posed by Kobayashi in 1970: It is well known that if M is a Hermitian manifold with negative holomorphic curvature, then M is Kobayashi hyperbolic. It was natural for Kobayashi to ask whether the converse is also true, i.e., does a Kobayashi hyperbolic manifold M admit a Hermitian metric of negative holomorphic curvature? In a joint project with G. P. Balakumar at ISI Chennai, I studied the Wu metric on convex and non-convex egg domains in $\mathbb{C}^n$. We verified that the Wu metric provides an affirmative answer to the above question of Kobayashi for these egg domains.

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Interplay between complex geometry and operator theory on two domains in $\mathbb{C}^3$

We have studied complex geometry and operator theory on polynomially convex and inhomogeneous domains in $\mathbb{C}^3$ namely the tetrablock and the symmetrized tridisc which in literature are denoted by $\mathbb{E}$ and Γ_3 respectively and are defined in the following way

$$\mathbb{E} = \{(x_1, x_2, x_3) \in \mathbb{C}^3 \, : \, 1 - zx_1 - wx_2 + zwx_3 \neq 0 \text{ whenever } |z| \leq 1, |w| \leq 1\}; \tag{2.1}$$

$$\Gamma_3 = \{(z_1 + z_2 + z_3, z_1z_2 + z_2z_3 + z_3z_1, z_1z_2z_3) : |z_i| \leq 1 \, i = 1, 2, 3\}. \tag{2.2}$$

The relevance of these two domains lie in its connection with the most difficult and appealing problem of μ-synthesis whose particular case is the classical Nevanlinna-Pick interpolation problem. We have done operator theory on these domains which will help the control engineers and geometers to find out solution to the μ-synthesis problem at least partially and hence the impact of this work is expected to be substantial.

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Extremal & probabilistic combinatorics

I work primarily in extremal and probabilistic combinatorics. Most problems of an extremal nature involve a set (finite) with certain constraints (which are of a combinatorial nature) and one is then interested in how large/how small such a family could be. In classical problems of extremal combinatorics, the extremal families also admit short (low complexity) descriptions. However, the more interesting ones are those where the class of extremal families is very large, without any seeming sense of structure, other than the fact that they are extremal families of a certain combinatorial extremal problem. Interesting instances of these occur in the study of graph coloring problems where one imposes different kinds of restrictions on the coloring.

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Scientific contributions by Prof. Sharad S. Sane

My mathematical work is broadly in the area of combinatorial configurations, finite geometries, graph theory and strongly regular graphs. Earlier work includes combinatorial characterizations of uniform projective Hjelmslev planes in terms of partially balanced designs with certain regularity conditions. It also includes constructions of a large number of new step parameter sequences for these objects. Uniform projective Hjelmslev planes were linked by me to the questions on cliques in uniform hypergraphs. This question was raised by Erd¨os. I studied geometric nets with deficiencies and the questions on extensions of finite projective planes. Characterization results also include those concerning embedding a combinatorial configuration obtained after removal of Baer subplanes from a projective plane. I looked at the question of Ramsey multiplicity of complete graphs in a 3-coloring of edges. A purely combinatorial proof of the non-existence of a biplane with block size 7 was given. Techniques are also developed towards a classification theory of quasi-symmetric designs. This classification theory began some decades ago with major papers of mine as the founding papers in this area, and are expected to pave way towards a proof of a conjecture which states that barring a few genuine exceptions (which are an outcome of some number theoretic facts), no other stipulated combinatorial objects are expected to exist. My work has applications in cryptography in terms of covering radius of a code.

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Designs, finite geometries and graphs

Focus of Prof. Singhi's research has been problems in theory of designs, finite geometries, codes, families of finite sets, graphs and hypergraphs etc. Algebraic methods are among the main tools, used for studying some of these combinatorial problems, in his papers.

In one of his first papers, he solved a problem initially posed by famous group theorist M. Hall, of classifying all (19, 9, 4) designs and their residual designs (designs coming from Hadamard Matrices of order 20). Later in a joint papers with R. C. Bose and S. S. Shrikhande, he solved a classical problem of embedding a residual design into a corresponding symmetric design when block sizes are sufficiently large λ (this corresponds to generalizing embedding of an affine space into a projective space to all symmetric designs with large block sizes). These results have been considered path breaking and a landmark by many experts.

Minimal forbidden graphs for generalized line graphs and related other families were characterized in his joint papers with S.B. Rao, K.S. Vijayan, G. R. Vijayakumar and D.R. Hughes. Also studied and characterized were some sharply edge transitive graphs in his joint work with P. J. Cameron, M. Deza and L. Babai. In joint papers with Erdos, Deza, Rothschild, and Frankl, some extremal set systems and perfect matroid designs were characterized. In a joint paper with Shrikhande, the well-known λ-design conjecture of H. J. Ryser was shown to be true for all prime λ. Their result is still among the best results known on this longstanding conjecture.

In a joint paper with Shrikhande, it was shown that the set of all t-designs on a given set of points and with a given block size, form a toroidal monoid. The corresponding Cohen Macaulay Rings and Poincare series were studied in this paper, to obtain a reciprocity relation on the number of t-designs. In his joint papers with Ray Chaudhuri the well known existence conjecture for t-designs was shown to be true for all large v and large λ. Again these and their sequels by the same authors generated a lot of activities in the area. Their methods also were algorithmic and algorithms based on their papers have been used to construct t-designs with large t.

In a joint paper with Ray-Chaudhuri, Sanyal and Subramanyan unidirectional error correcting codes used in computer memory chips and other electronic instruments were studied, in several cases best possible such codes were described. In a joint paper with T. A. Anthony and S. Iyengar, Hypercube architecture for inter connection network was studied, using a theorem of Klietman and best fault tolerant partitions were described in practical cases. Singhi has been recently developing a general theory for semiadditive rings. These are very general type of rings closely related with planar ternary rings of projective planes. A free such ring generated by 1 can be thought of as a ring of generalized integers (without associativity, commutativity, linearity etc.). A research paper recently published by has created a lot of interest in this area with the hope that the methods developed by him to study these rings will develop a strong tool to study the well-known prime power conjecture for planes. The prime power conjecture is considered to be one of the basic problems in Mathematics and has remained unsolved for more than 100 years. He has published more than 75 research papers.

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Combinatorics

I work primarily in enumerative combinatorics and linear algebraic graph theory with a strong bias towards q-analogues. I am also interested in the enumerative aspects of posets, graphs and polytopes. In linear algebraic graph theory, I am interested in getting graph theoretic information by applying linear algebraic methods to matrices associated with graphs. In particular, I have been studying properties of various distance matrices and laplacian matrices associated to graphs.

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Enumerative and algebraic combinatorics

My initial work was in Combinatorial optimization and Sperner theory. Over the last several years I have been primarily interested in explicit diagonalization and block diagonalization of several natural *-algebras of matrices occurring in combinatorics.

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Computational complexity and pseudorandomness

The goal of a Theoretical Computer Scientist is to understand the power of computation: can computers perform the tasks one is interested in? Can they do so efficiently, with constraints on resources such as time, space, non-determinism, parallelism, randomness, etc.? The "right" constraint might depend on the application at hand: algorithmists often want linear-time algorithms for their problems; logicians are sometimes satisfied to prove that their algorithms halt in finite time; complexity theorists of different flavours look at various notions of efficiency.

My own work in has primarily been in Circuit Complexity, where the resource in question is the size of the smallest circuit computing a given function, which is closely related to the time required to compute the given function. I am mostly interested in circuits of two flavours: Arithmetic circuits, which are made up of gates that perform arithmetic operations (such as addition, multiplication, etc.) over a fixed field and Boolean circuits, which are made up of logical gates (such as AND, OR, etc.) and compute Boolean functions. The challenge always is to design "explicit" lower bounds: it is known that "most" functions require large circuits, and the challenge is to find one. This kind of challenge is quite common in Theoretical Computer Science and Combinatorics and goes by the title of Pseudorandomness, which is also a major theme of my research. I describe it below.

Many interesting computational and combinatorial objects, such as Error-Correcting Codes, Expander graphs, and others, can easily be shown to exist via probabilisitic arguments. An interesting computational challenge is to match these constructions with explicit ones. Quite apart from the fact that such questions yield very interesting mathematics, a concrete motivation for these questions comes from the question of whether probabilistic algorithms are significantly more efficient than deterministic (i.e. non-probabilistic) algorithms. The research community believes that the answer is yes, but the way forward involves showing how to circumvent probabilistic existence arguments with deterministic ones.

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Geometric Langlands duality and Higgs bundles

The classical limit of the geometric Langlands duality concerns Hitchin systems in the following way see Beilinson and Drinfeld [B-D], Arinkin and Fedorov [A-F]. The ground field will be $\mathbb{C}$. Let G be a semisimple group, and let X be a compact Riemann surface. Let $\mathcal{B}un_G$ be the moduli stack of principal G-bundles on X. Then the cotangent stack $T^*\mathcal{B}un_G$ can be identified with the stack of Higgs bundles $\mathcal{H}iggs_G$ on X. This stack is a local complete intersection. Using Ad-invariant polynomials, one can define the Hitchin fibration

$$p : \mathcal{H}iggs_G \to H_G = \oplus_{i=1}^{k} H^0(X, K_X^{d_i}).$$

At the level of coarse moduli spaces, this is flat and projective, and the generic fibres are torsors under abelian varieties see Donagi and Pantev [D-P]. Let $\check{G}$ be the Langlands dual to G. Identifying H_G with $H_{\check{G}}$, one can consider the 'dual' Hitchin system $\check{p} : \mathcal{H}iggs_{\check{G}} \to H_G$. In the so called 'classical limit', the geometric Langlands duality for Hitchin system conjectures that there is a coherent sheaf $\tilde{L}$ on $\mathcal{H}iggs_{\check{G}} \times_{H_G} \mathcal{H}iggs_G$ such that the corresponding integral transform

$$D(Higgs_{\check{G}}) \to D(Higgs_G) : \mathcal{F} \mapsto Rp_{2,*}(\tilde{L} \otimes^L p_{1,*}\mathcal{F})$$

is an equivalence of categories. We do not expect this to hold literally but we expect this to hold over a big open subspace of the Hitchin base H_G. Generically over the base, the conjecture is settled by Donagi and Pantev see [D-P], [A-F].

(1) Dima Arinkin and Roman Fedorov see [A-F] considered a slightly different but a more general situation. They considered a degenerate family of abelian varieties $X \to B$, and considered the 'dual' family $\mathrm{Pic}^\tau(X/B)$. They constructed a partial Fourier-Mukai type transform, and showed that there is an embedding

$$D^{(-1)}(Pic^\tau(X/B)) \to D(X) : \mathcal{F} \mapsto Rp_{2,*}(\mathcal{L} \otimes p_1^*\mathcal{F}),$$

where $D^{(-1)}(Pic^\tau(X/B))$ is a particular full subcategory of $D(\mathcal{P}ic^\tau(X/B))$.
In a sense the results of Arinkin and Fedorov give only a "partial" Fourier-Mukai transform. I would like to prove a stronger result about Fourier-Mukai transform in the context of integrable systems (or possibly in a more specific context).

(2) Parabolic bundles can be regarded as bundles on a Deligne-Mumford stack see Biswas [B], where bundles on Deligne-Mumford stacks are referred to as orbifold bundles. I would like to extend the results of Donagi and Pantev to the parabolic setting. Some progress has already been made by Indranil Biswas and Arijit Dey, but that is restricted to the case of parabolic SL_n-bundles, while they considered only full parabolic flags in order to get a smooth generic spectral curve.

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Dynamics and diophantine approximation & exponentiality and root problems in groups

I have made major contributions to the study of the dynamical behavior of flows on homogeneous spaces of Lie groups and applications of the theory to problems in Diophantine approximation. One of the problems concerns the set of values of quadratic forms, evaluated at points with integer coordinates. Conditions for the set to be dense, and estimates for the number of solutions with values in prescribed intervals, have now been understood, through the work of various authors including myself, in the case when the number of variables is at least 3. In the case when there are only two variables, viz. binary quadratic forms, different techniques are needed to understand the corresponding issues, and it has been the subject of my research work in recent years. The work involves continued fraction expansions of numbers. In this context, various ways of producing continued fraction expansions for real as well as for complex numbers have been studied in my recent papers, some in collaboration with others. The work broadens the scope of the theory, and is in particular applied to the study of values of binary quadratic forms. Using continued fraction expansions in conjunction with the study of the geodesic flow associated with the so called modular surface, asymptotic estimates are given for the number of solutions, in integer pairs, for binary quadratic forms to take small values.

Work was also done recently on the issue of surjectivity of the exponential maps of Lie groups, and that of the n th power maps of a large class of groups (equivalently, existence of n th roots in groups), the latter in collaboration with Arunava Mandal, my student at the Department. Conditions are described for large subsets in certain solvable groups to admit n-th roots.

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Enumerating nodal plane curves in P³

In a current project with Ritwik Mukherjee and Martijn Kool, we attempt to prove a variant of the Gottsche conjecture by looking at the number of δ-nodal degree d planar curves in $\mathbb{P}^3$ intersecting the right number of generic lines. We wish to show that for $d \geq \delta$ this number is given by a universal polynomial of some determined degree involving d. We attempt to do this by performing a relative version over the Grassmannian $Gr(3,4) \cong \mathbb{P}^{3*}$ of the computations of Kool-Thomas-Shende in their proof of the Gottsche conjecture. We also wish to verify our answers by comparing to direct counts using different methods developed by Ritwik and Zinger and possibly also comparing our results to certain Gromov-Witten calculations by Pandharipande. Perhaps a more ambitious aim will be to also consider suitable generalisations to planar curves in higher dimensional projective spaces.

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Hopf monoids in species

Joyal's species constitute a good framework for the study of algebraic structures associated to combinatorial objects. In joint work with Marcelo Aguiar, we studied the category of species focusing particularly on the Hopf monoids therein. The notion of a Hopf monoid in species parallels that of a Hopf algebra and reflects the manner in which combinatorial structures compose and decompose. We constructed examples from combinatorial and geometric data inspired by ideas of Rota and Tits' theory of Coxeter complexes.

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Homotopy theory in the category of graphs

My research interests lie in stable homotopy theory and topological combinatorics. Computing good lower bounds for chromatic number of graphs is an important problem in combinatorics. Kozlov, Babson, Dochtermann and several others have proved results in this context which hinge on understanding the connectivity of certain simplicial complexes called Hom complexes. Several notions from homotopy theory of topological spaces like homotopy equivalences, etc. have analogues in the category of graphs. In joint work with Shuchita Goyal, we are working toward understanding appropriate model structures on the category of graphs which give a better understanding of Hom complexes and allow us to apply other tools from homotopy theory in graphs.

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Special values of L-functions

Generalizing Euler's classical theorem on the values $\zeta(2k)$ of the Euler-Riemann ζ-function at positive even integers $s = 2k$, Deligne has stated a far-reaching conjecture about the behaviour of motivic L-functions $L(s, M)$ at their critical points $s = k \in \mathbb{Z}$. Deligne's conjectured formula expresses the critical L-values in question up to multiplication by elements in a concrete number-field $E(M)$, depending on the motive M, in terms of certain geometric period-invariants $c^{\pm}(M)$, as well as certain explicit integral powers $(2\pi i)^{d(k)}$:

$$L(k, M) \in (2\pi i)^{d(k)} c^{(-1)^k}(M) \cdot E(M).$$

Current problems I am working on deal with proving algebraicity results for special values of L-functions. Such algebraicity results were conjectured by Deligne for L-functions arising in algebraic geometry. Using the well known conjectural dictionary between L-functions arising from algebraic geometry and those arising from automorphic forms, it is an interesting question to ask if one can prove theorems for automorphic L-functions, of the kind predicted by Deligne.

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Hyper geometric monodromy groups: Arithmetic or thin?

The monodromy groups of the n-th order hypergeometric differential equations are characterized as the subgroups of the general linear group $GL(n)$, which are generated by the companion matrices of two monic co-prime polynomials $f(x)$ and $g(x)$ of degree n. If we impose some conditions on the polynomials $f(x)$, $g(x)$ (like $f(x)$ and $g(x)$ are not simultaneously polynomials in higher powers of x, and they are self-reciprocal), then the associated monodromy group preserves either a non-degenerate symplectic form (this happens only when n is even, and $f(0) = g(0) = 1$) or a non-degenerate quadratic form (and this happens when $f(0) = 1, g(0) = -1$), and it is contained in the respective symplectic group (of the corresponding symplectic form) or in the orthogonal group (of the corresponding quadratic form) as a Zariski dense subgroup.

In addition to the ongoing conditions required for the monodromy groups being a Zariski dense subgroups of the respective symplectic or orthogonal groups if we also assume that the polynomials $f(x)$ and $g(x)$ have the integer coefficients, then the associated monodromy groups, being generated by the companion matrices of two monic polynomials having integer coefficients with the constant terms +1 or -1, are subgroups of the integral symplectic or integral orthogonal groups. When the monodromy group is of finite index in the corresponding integral symplectic or integral orthogonal groups, it is called ARITHMETIC, and THIN otherwise.

My research so far have been around proving these hypergeometric monodromy groups arithmetic. For example, in a joint work with T. N. Venkataramana (TIFR Mumbai), we obtained that whenever n is even, and the constant terms of the polynomials $f(x)$ and $g(x)$ are +1, and the absolute value of the leading coefficient of the difference polynomial $f(x) - g(x)$ is less or equal to 2, then the associated monodromy group is arithmetic. I have also shown (in a further article) the arithmeticity of some hypergeometric monodromy groups which arise from Geometry, and I am now trying to show the arithmeticity or the thinness of the remaining groups.

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Hyperbolic partial differential equations and applications

My research interest lies in the field of linear and nonlinear wave propagation. More specifically, my interest is in the study of partial differential equations of hyperbolic type. Most of the systems of PDEs (of practical interest) are not solvable exactly to get closed form solutions and therefore an approximation procedure needs to be adopted. Also, since the solution spaces of these systems are often vast (for instance the space of BV functions in the case of hyperbolic system of conservation laws), any qualitative analysis becomes more challenging. Because of the availability of powerful computational technology, numerical study of such problems attracted many researchers. However, in some practical applications, it is very expensive to go for full numerical simulations. Hence, in order to minimize the computational cost and/or to bring out some specific features of the solution, certain approximations of a governing system are in focus. My research focuses on developing some robust numerical methods and other methods based on nonlinear geometric theory, and their analysis.

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Finite element methods for plate bending problems

Plates are plane structural elements with a small thickness compared to the planar dimensions. Depending on the length to thickness ratios, plates are classified into moderately thick, thin and very thin plates. One of my primary research interests has been to employ finite element methods to determine approximations to deformation and stresses in thin and very thin plates when they are subject to loads. A Kirchoff model is used for the *thin* elastic plates and this leads to fourth order elliptic equations with the transverse displacement as the unknown variable. For *very thin* plates, a model proposed by von Kármán leads to a fourth order semi-linear system of partial differential equations with transverse displacement and stress variable as unknowns. My research work focusses on developing *a priori* and *a posteriori* error estimates for conforming, nonconforming, discontinuous Galerkin and mixed finite element approximations to fourth order elliptic equations and nonsingular solutions of von Kármán equations. Another area of interest is to study *a priori* and *a posteriori* finite element methods for distributed optimal control problems governed by thin and very thin plates.

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Partial differential equations, completely integrable systems and special functions

(1) *Partial Differential Equations, Shock waves in Hyperbolic Systems of Conservation Laws.*

- Structure of WTC expansions. This is related to completely integrable systems of PDEs. It has been observed that a large number of PDEs that are completely integrable via the inverse scattering transform also pass the so called WTC test which means the solutions of the differential equations in the complex domain are devoid of movable branch points. Since a perturbation of a completely integrable system may fail to remain so, it is of interest to investigate the structure of the logarithemic series solutions. The matter had remained somewhat vague until we discovered a surprising connection with the representation theory of SL2(C) in the space of binary forms.

- The theory of hyperbolic conservation laws is anothoer component of my research in PDEs where we have looked at the propagation of high frequency waves and modulation problems. In one case the idea of WTC expansions have been applied to study the case of imploding shells. The differential equations considered arise mostly from fluid flow.

(2) *Dynamical systems* This comes from the theory of completely integrable systems as well as the theory of fluid flow. The connection with the former is too well known. ODE reductions of the governing equations of fluid flow in stratified environments such as planetary atmospheres taking into account the effect of rotations and Coriolis forces leads to interesting systems of ordinary differential equations that can be investigated via standard techniques of dynamical systems - specifically the location of periodic orbits and their stability via Floquet analysis.

(3) *Classical Analysis, Special Functions and History of Mathematics.*
My main interest is in the study of the Gamma function which plays a central role in the theory of special functions and orthogonal polynomials. There has been a resurgent interest in this field in view of the surprising connections with random matrix theory and quantum groups.

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Research undertaken by Prof. V. D. Sharma

For the last couple of years, we have been working on the asymptotic solutions of Navier - Stokes system of PDEs (in one- and two- dimensions) governing the propagation of weakly nonlinear waves in non-ideal (real) fluids. The main focus of our study has been to explore, analytically and numerically, the real gas effects and the manner in which they influence the waves with regard to their speed, strength, the on-set of interaction, and decay behavior. The problems involving the reflection-diffraction of weak shocks and that of expansion of a wedge into vacuum have also been studied within the context of real gas effects using asymptotic methods, hodograph transformations, and solutions of the Riemann problem; the work has culminated into Couple of publications:

Presently, we are working on the interaction/ propagation of nonlinear waves governed by a Hall-MHD system of PDEs involving the viscosity, resistivity, and the Hall effect; the evolution equation using reductive perturbation approach involves quadratic and cubic nonlinearities along with dissipative and dispersive effects. Efforts to look for analytical and numerical solutions of this problem are currently under way.

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Asymptotic analysis of PDES, conservations laws

(1) *Asymptotic analysis of PDEs:* Most of the physical phenomena are modeled by Hyperbolic PDEs and in many of them feature multiple characteristics. Roughly it corresponds to studying crossings of eigenvalues of operators parametrized by a vectorial parameter. We are studying some of such situations.

(2) *Conservation laws:* We are studying the question of convergence generalized viscous approximations to entropy solutions of hyperbolic conservation laws.

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Innovative applications of statistical data mining in computational molecular biology & medical diagnostics

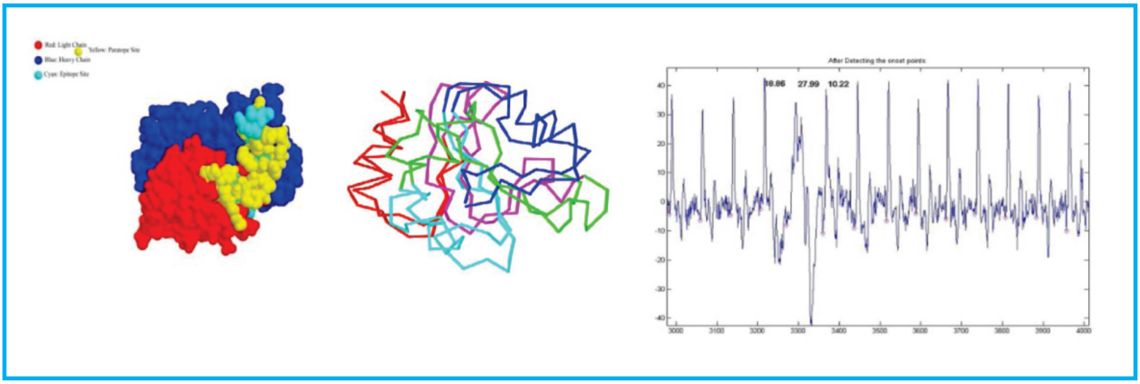
Extension of Recent Contributions for Wider Applications in Computational Biology/Drug Discovery/Immunoinformatics/Automated Pulse-Diagnostics

¶ ParaDes: AI Software for Epitope-Paratope Designing (Copyright: GoI No. SW-698/2002) deploying knowledge-based correlation mapping on Hopfield Network.

¶ Efficient ab-initio Protein Structure Prediction using AI & Nonparametric Statistics. [Web-server: www.math.iitb.ac.in/~epropainor/]

¶ Efficient ab-initio Protein Structure Prediction using AI & Nonparametric Statistics. [Web-server: www.math.iitb.ac.in/~epropainor/]

¶ Automated Diagnosis of Cardiac Problems, Diabetes & Other Diseases using Time-Series Analysis and Geometric Modeling of IPG of Radial Pulse.



- Developed new algorithm for *Multidimensional Markov Chain Monte Carlo* (MCMC). < Tests on bench-mark samples & real data ongoing >.

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Statistical inference in multi-state coherent systems

My research area is Reliability Theory. Currently, I have been working on research problems that deal with Multi-state coherent systems. These systems are such that their components and systems themselves can be in one of $(M+1)$ possible states $0, 1, 2, \cdots, M$ at any given time, where the extreme states 0 and M represent completely failed and completely working states respectively, and others are intermediate states that decrease in performance level as the system or a component makes a transition from state i to $(i-1)$, $i = M, (M-1), \cdots, 1$. The specific research problems related to Multi-state coherent systems I have been working on are given below:

(i) Estimation: Suppose that the data are in terms of $\min_i T_{ik}$ and $\max_i T_{ik}$ where T_{ik} is the amount of time the ith component spends in state k for $i = 1, 2, \cdots, n$ and $k = 1, 2$, and a random vector (T_{i1}, T_{i2}) is positively quadrant dependent (PQD) for each $i = 1, 2, \cdots, n$. It is worth noting that parameter estimation using maximum likelihood method even for $n = 2$, a particular PQD distribution Farlie-Gumbel-Morgenstern with non-identical exponential marginal distribution functions, and for standard coherent systems such as series and parallel systems becomes computationally extremely challenging.

(ii) Construction of Reliability Test Plans: Optimal reliability test plans for Multi-state coherent systems of type I and Multi-state systems of type II (also referred to as Binary Type Multi-state coherent Systems) have been proposed. These test plans have been demonstrated using standard Multi-state systems such as Series and Parallel systems.

(iii) Degradation Modelling: There are numerous instances in which data are available in the form of degradation rates or times as opposed to failure rates or times. A problem involving modeling of degradation data for Multi-state coherent systems is being investigated.

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Patterned random matrices

My research interests can be broadly classified into three areas, namely, random matrices, free probability theory and statistics. Study of random matrices, more precisely the properties of their eigenvalues, has emerged first from data analysis and then from statistical modelling for heavy nuclei atoms. A mathematical theory of the spectrum of the random matrices began to emerge with the work of E. P. Wigner, F. J. Dyson, M. L. Mehta, C. E. Porter and co-workers in the 1960s. One of the important object of study in random matrix theory is the asymptotic behaviour of the eigenvalues in bulk when the dimension of the matrix goes to infinity. We try to answer this question when the structure of the matrices follow a specific pattern. We also study the fluctuation of linear statistics of eigenvalues of random matrices as dimension goes to infinity where the linear statistics of eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of A_n corresponding to a continuous test function f is defined as

$$\sum_{k=1}^{n} f(\lambda_k).$$

Free probability theory has a deep connection with the random matrices. The asymptotic behaviour of the empirical distribution functions of a collection of random matrices are connected to different notions of independence in free probability theory. We try to explore these notions of independence for different patterned matrix models and study the limiting behaviour of patterned matrices using free probability techniques. Although, many results obtained for free probability are quite similar to the results known in classical probability theory, there are exceptions as well. Further, due to its non-commutative structure, the study of free probability is challenging and interesting. In statistics, we study the limiting behaviour of time series models, in particular, the regenerative property of auto regressive models with random coefficients and their limiting behaviour. We also plan to study multivariate time series models when the coefficients are random matrices of fixed dimension.

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Statistical analysis of random processes

My research interest primarily lies in understanding the statistical behavior in random phenomenon and its modelling. My broad research areas are time series analysis, spectrum estimation, non-parametric regression and high-dimensional data analysis.

A continuous time second order stationary stochastic process is characterized by its mean and spectrum. One often infers about spectrum of the process based on discrete sample of it. Thus, sampling of continuous time process plays an important role in the statistical inference of spectrum of the process. It is well known that if the spectrum of such a process is not bandlimited, then uniform sampling of the process cannot identify spectrum of the continuous time process. This problem is referred to as aliasing. Several stochastic point process sampling schemes, for example Poisson point process, turns out to be alias free sampling scheme and leads to consistent estimation of the spectrum. However, we observe that large sample asymptotic property such as consistency of smoothed periodogram spectrum estimator can be established based on uniform sampling if the sampling rate increases indefinitely. Further, spectrum estimator based on stochastic point process sampling exhibits larger variance than that of based on uniform sampling and the corresponding biases are in reverse direction. It also turns out that mean square error of the estimator based on stochastic sampling do not have clear dominance over that of based on uniform sampling.

On the other hand, alias-free stochastic sampling schemes have an inherent advantage of having large number of successive samples that are less than any threshold in contrast to uniform sampling scheme. If a hard limit is imposed on the sampling scheme in terms of threshold for separation between successive samples, then no sampling scheme is alias-free for the class of non-bandlimited spectrum. Among the class of bandlimited spectrum, stationary renewal process sampling scheme are alias-free for a class of larger spectrum support than that of uniform sampling scheme even under the said hard threshold and consistent estimator of spectrum is also provided.

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Stochastic process and application to control theory

My area of research is broadly in stochastic processes and its applications. At present, I am working on two type of problems, namely risk-sensitive control theory and small noise perturbations of dynamical systems.

In risk-sensitive control problems, we investigate existence of optimal risk-sensitive control under various state dynamics. We have some partial results on the existence of risk-sensitive optimal control under geometric ergodicity assumptions. Also we have established a connection between Collatz-Wielandt formula and risk-sensitive value when state dynamics evolves in a smooth bounded domain or when the state dynamics is given by stochastic differential equations with periodic coefficients.

In small noise perturbation of dynamical systems, we study effects of small noise limit to the invariant density of the dynamical system. We have studied the small noise asymptotics of the invariant density for the normalized Feynman - Kac semi group.

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Collapsibility of Association measures

Collapsibility deals with the conditions under which a conditional (on a covariate W) measure of association between two random variables Y and X equals the marginal measure of association. We have discussed the average collapsibility of certain well-known measures of association, and also with respect to a new measure of association. The concept of average collapsibility is more general than collapsibility, and requires that the conditional average of an association measure equals the corresponding marginal measure. Sufficient conditions for the average collapsibility of the association measures are obtained. Some interesting counter-examples are constructed and applications to linear, Poisson, logistic and negative binomial regression models are discussed. An extension to the case of multivariate covariate W is also analyzed. The collapsibility conditions of some dependence measures for survival models are also investigated and illustrated them for the case of linear transformation models.

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