Maths meets physics

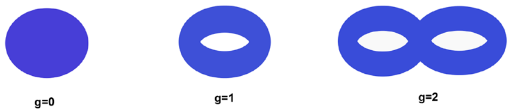

Figure 1: Two dimensional surfaces with genus g

Trefoil knot : 3_1 Figure eight knot : 4_1 5_1 5_2

Newton's laws of motion are based on *calculus*. Einstein unified gravity with Special Theory of Relativity using *Riemannian geometry*. We see that mathematics provides a language by means of which physical laws can be formulated to explain nature. Einstein's theory of General Relativity inspired developments in the area of *Differential Geometry*. This interplay of mathematics applied to physics and physics inspiring development in mathematics has intensified in the last five decades.

Topology of two dimensional surfaces is completely classified using genus and punctures on the surfaces (Fig.1). However complete classification of the three dimensional manifolds is still an open problem. In fact, even a simpler problem of *classification of knots* (non-intersecting curves in three manifolds drawn in Fig.2), is a challenging question for both mathematicians and physicists. Jones has introduced a skein / recursive method of writing down polynomials in variable q for knots but the physical meaning of the variable q is unknown.

Our research focus

In late 1980s, the study of mathematical questions of *topology* and *geometry* of low dimensional manifolds were addressed using principles of quantum physics, namely, *topological quantum field theories*. Particularly, *Chern-Simons theory*, a topological field theory, provides natural framework for the study of knots and three manifolds. Witten's pioneering work reproduced Jones' polynomial and gave meaning to the polynomial variable q. Interestingly, this framework led to tabulating many generalised invariants (sometimes also known as colored invariants) for the knots. It appears that physically motivated theory such as *Chern-Simons theory* can solve classification problem of knots.

The polynomial form of colored invariants for arbitrary colors can be written down only if we know duality matrix elements. The properties of the duality matrix are obeyed by Wigner 6j or Racah coefficients. Even though we managed to write these matrix elements for some class of colors, we are still trying to write closed form expression for arbitrary colors. If we succeed, we could tabulate colored polynomials for all colors and address the classification problem of knots.

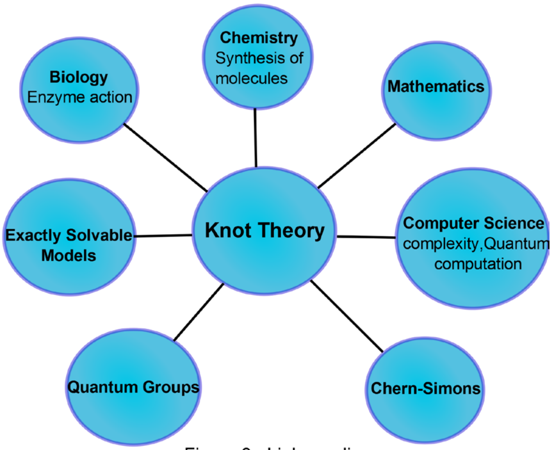

Figure 3 : Linkage diagram

String theory, a promising candidate for unifying all four fundamental forces, introduced a concept of duality which relates two apparently different theories describing the same physical system. When one theory is weakly interacting and is readily solvable, the other theory could be strongly interacting and difficult to solve. As put forth by Maldacena in 1997, famously known as AdS-CFT correspondence, closed string theory in anti-de Sitter background is dual to maximally supersymmetric Yang-Mills theory. In the same spirit, Gopakumar and Vafa conjectured that Chern-Simons theory is dual to a closed topological string theory. This has led to a flurry of interesting conjectures on relating quantities on the string side with colored invariants of knots. Specifically, the latter can be used to determine the number of stable states in string theory which is otherwise difficult to determine within string theory.

Our group at IIT Bombay has contributed to development of the colored invariants. We were among the first to establish the polynomial form of such invariants for some colors conjecturing closed form expression for quantum Racah coefficients. We are in the process of developing the Knotebook website with Russian collaborators to enumerate colored invariants for many colors. Our results play a crucial role in deducing the number of stable states in string theory. Our work is thus a progress towards complete classification of colored knots. We hope to achieve colored invariants for arbitrary colors soon.

Linkages and applications

In broad perspective, knot theory is one of the research areas attracting interdisciplinary and intra disciplinary collaboration and interaction. We have been actively working on the mathematics and physics interface of knot theory. We strongly believe that there could be exciting interdisciplinary collaboration with biologists studying knotting structures of enzyme action at synaptic nodes and chemists addressing stereo isomers of molecules in the forthcoming years.

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